International Journal of Theoretical Physics, Vol. 42, No. 3, March 2003 (© 2003)

Quantum Properties of Odd Excited Negative Binomial States of the Radiation Field

H. H. Salah,[1] M. Darwish,[2] and A.-S. F. Obada[3,4]

Received November 11, 2002

The odd excited negative binomial states are introduced using the photon creation operator by repeated application on negative binomial states. These states interpolate between the odd displaced Fock states and the odd excited pure thermal states. In this paper both squeezing phenomena (normal squeezing and amplitude squared squeezing) are discussed. Besides discussion of the Glauber second-order correlation function, investigations are carried out for the quasi-probability distribution functions (Wigner function and Q-function). Finally the Pegg–Barnett phase probability distribution is computed for these states.

KEY WORDS: negative binomial states; radiation field.

1. INTRODUCTION

The importance of nonclassical states in different branches of physics (in particular quantum optics) hardly needs to be emphasized. Recently there has been considerable interest in generating and producing new quantum states in addition to the number state (Fock state) $|n\rangle$ and the coherent state $|\alpha\rangle$. Most of these states are intermediate field states, which interpolate between distinctive states, reducing to them in certain variation of limits of the parameters involved. For example, Stoler *et al.* (1995) introduced the binomial state $|\eta, M\rangle$, which is a linear combination of M states with coefficients chosen such that the photon-counting probability distribution is binomial with mean photon η, M. This state is intermediate between the number state and the the coherent state. Also, we see that the negative binomial state has the attractive feature that in limiting cases it corresponds to fields in coherent and pure thermal states (Agarwal, 1992; Agarwal and Inguva, 1991; Joshi and Lawande, 1991). Another example we could mention here is the logarithmic

[1] Faculty of Science for girls, Al-Azhar University, Nasr City, Cairo, Egypt.
[2] Faculty of Education, Suez Canal University at Al-Arish, Egypt.
[3] Faculty of Science, Al-Azhar University, Nasr City, Cairo, Egypt.
[4] To whom correspondence should be addressed at Faculty of Science, Al-Azhar University, Nasr City 11884, Cairo, Egypt; e-mail: obada@malier.scu.eun.eg.

state, which has been investigated with regard to the nonclassical character of radiation fields (Mahran and Obada, 1988; Simon and Satyanarayana, 1988). This state is a special case of the negative binomial distribution with the terms $n = 0$ removed (Kendall and Stuart, 1977). As another example, it would be interesting to refer to the generalized geometric state, which represents the gradual behavior of some quantum optical systems where the state of the radiation field changes from the number state $|n\rangle$ to the (nonpure) chaotic state (Batarfi *et al.*, 1995; Obada *et al.*, 1993). Furthermore, the even (odd) negative binomial states (Joshi and Obada, 1997), interpolate between the even (odd) coherent states and the even (odd) quasi-thermal states. Recently, the excited binomial states (EBS) and excited negative binomial states (ENBS) (Wang and Fu, 2000) bridge between coherent states and Fock states in one and excited thermal states in the other.

The aim in this paper is to introduce and study the odd excited negative binomial states (OENBS). Such state can be generated by repeated application of the photon creation operator on the odd or even negative binomial states (NBS) (Barnett, 1998; Fu and Sasaki, 1997a,b; Matsuo, 1990; Srinivasan and Lee, 1996; Vourdas and Bishop, 1995). The characteristics of OENBS are studied in the following sequel: the nonclassical properties such as squeezing phenomena [normal squeezing and amplitude squared squeezing (ASS)] and the Glauber second-order correlation function are studied in Section 3. The quasi-probability distribution functions (Wigner function and Q-function) are studied in Section 4. The phase properties in the Pegg–Barnett formalism is considered in Section 5. Finally the conclusions are summed up in Section 6.

2. THE NORMALIZED OENBS

The ENBS of the radiation field are introduce by repeated application of the photon creation operator on NBS $|\eta, M\rangle^-$ as defined in Wang and Fu (2000) by the following:

$$|k, \eta, M\rangle^- = N^-(k, \eta, M)\hat{a}^{\dagger k}|\eta, M\rangle^- \tag{1}$$

$$= N^-(k, \eta, M)\sum_{n=0}^{\infty} B_n^M(k)|n+k\rangle \tag{2}$$

where $N^-(k, \eta, M)$ is the normalization constant while the NBS is defined by,

$$|\eta, M\rangle^- = \sum_{n=0}^{\infty}\left[\binom{M+n-1}{n}\eta^{2n}(1-|\eta|^2)^M\right]^{1/2}|n\rangle \tag{3}$$

and

$$B_n^M(k) = \left[\binom{M+n-1}{n}\eta^{2n}(1-|\eta|^2)^M\frac{(n+k)!}{n!}\right]^{1/2} \tag{4}$$

is the photons amplitude of ENBS, while $|n\rangle$ is the usual number state, the interval of the parameter η is $0 \le |\eta| \le 1$, M is a nonnegative integer, while the boson creation $\hat{a}^\dagger$ and annihilation $\hat{a}$ operators satisfy $[\hat{a}, \hat{a}^\dagger] = 1$.

The construction of the OENBS is based on the definition of ENBS of Eq. (2), such that the states $|n+k\rangle$ are always odd.

Therefore it can be rewritten as follows for two cases:

(i) even k, i.e., $k = 2k_1$

$$|2k_1, \eta, M\rangle_o^- = N_o^- \sum_{n=0}^{\infty} B_{2n+1}^M(2k_1)|2n + 2k_1 + 1\rangle^- \tag{5a}$$

(ii) odd k, i.e., $k = 2k_2 + 1$

$$|2k_2 + 1, \eta, M\rangle_o^- = N_o'^- \sum_{n=0}^{\infty} B_{2s}^M(2k_2 + 1)|2s + 2k_2 + 1\rangle^- \tag{5b}$$

where $B_{2n+1}^M(2k_1)$ and $B_{2s}^M(2k_2+1)$ are given by Eq. (4). The normalization constants of OENBS for even and odd k are N_o^- and $N_o'^-$, which are given respectively by

$$\left|N_o^-\right|^{-2} = \sum_{n=0}^{\infty} \left|B_{2n+1}^M(2k_1)\right|^2 \tag{6a}$$

$$\left|N_o'^-\right|^{-2} = \sum_{s=0}^{\infty} \left|B_{2s}^M(2k_2 + 1)\right|^2 \tag{6b}$$

It is known that, the ENBS tends to the ECS if we take the limit $\eta \to 0$ and $M \to \infty$ such that $M|\eta|^2 = |\alpha|^2$. In the other limiting condition, i.e., $\eta \to 0$, the NBS reduce to the vacuum state and the ENBS reduce to the Fock state $|k\rangle$. When $M = 1$ the quasi-thermal states are produced (Joshi and Obada, 1997). We note here that the Fock states $|0\rangle, |1\rangle, \ldots, |k-1\rangle$ are removed from the space as well as all the even number states.

In order to investigate the statistical characteristics of these states we calculate the expectation values for various operators. The mean photon number is defined as the expectation value of the number operator $\hat{n} = \hat{a}^\dagger\hat{a}$. It is easy to show that the expression of mean photon number $\langle\hat{n}\rangle_o$ in the OENBS for even and odd k is given respectively by,

$$\begin{aligned} {}^-\langle 2k_1, \eta, M|\hat{\eta}|2k_1, \eta, M\rangle_o^- &= \left|N_o^-\right|^{-2} \sum_{n=0}^{\infty}(2n + 2k_1 + 1) \\ &\quad \times \left|B_{2n+1}^M(2k_1)\right|^2 \end{aligned} \tag{7a}$$

$$\begin{aligned} {}^-\langle 2K_2 + 1, \eta, M|\hat{\eta}|2k_2 + 1, \eta, M\rangle_o^- &= \left|N_o'^-\right|^{-2} \sum_{s=0}(2s + 2k_2 + 1) \\ &\quad \times \left|B_{2s}^M(2K_2 + 1)\right|^2 \end{aligned} \tag{7b}$$

Similarly we can calculate the expectation value for the operator $\hat{n}^2$ as follows,

(i) for even k

$$\langle \hat{n}^2 \rangle_o = \left| N_o^- \right|^2 \sum_{n=0}^{\infty} (2n + 2k_1 + 1)^2 \left| B_{2n+1}^M (2k_1) \right|^2 \tag{8a}$$

(ii) for odd k

$$\langle \hat{n}^2 \rangle_o = \left| N_o'^- \right|^2 \sum_{s=0}^{\infty} (2s + 2k_2 + 1)^2 \left| B_{2s}^M (2k_2 + 1) \right|^2 \tag{8b}$$

Finally the expectation values of the powers of $\hat{a}$ are given by,

(i) for even k

$$\begin{aligned} {}_0^-\langle 2k_1, \eta, M | \hat{a}^m | 2k_1, \eta, M \rangle_0^- &= \left| N_o^- \right|^2 \eta^m (1 - |\eta|^2)^M \\ &\times \sum_{n=[m/2]}^{\infty} \frac{(M + 2n - m)!(2n + 2K_1 + 1)!}{(M-1)!(2n+1)!(2n - m + 1)!} \\ &\times \sqrt{\frac{(M+2n)!}{(M+2n-m)!}} |\eta|^{2(2n-m+1)} \end{aligned} \tag{9a}$$

(ii) for odd k

$$\begin{aligned} {}_0^-\langle 2k_2 + 1, \eta, M | \hat{a}^m | 2k_2 + 1, \eta, M \rangle_o^- &= \left| N_o'^- \right|^2 \eta^m (1 - |\eta|^2)^M \\ &\times \sum_{s=[m/2]}^{\infty} \frac{(M + 2s - m - 1)!(2s + 2K_2 + 1)!}{(M-1)!(2s)!(2s - m)!} \\ &\times \sqrt{\frac{(M+2s-1)!}{(M+2s-m-1)!}} |\eta|^{2(2s-m)} \end{aligned} \tag{9b}$$

For the expectation values of $(\hat{a}^\dagger)^m$, we take the complex conjugate of the above equations. It is clear that the expectation values of any odd power of the operators $\hat{a}$ and $\hat{a}^\dagger$ vanish. These results will enable us to discuss different statistical aspects of the states under investigation.

3. NONCLASSICAL EFFECTS

In this section we use the results obtained in the previous section to discuss the squeezing phenomena as well as the autocorrelation function.

3.1. Quadrature Squeezing

Squeezing is one of the most important phenomena in quantum optics because of its applications in various areas, e.g., in optics communication, gravitational wave detection, quantum information theory, etc. (Braunstein and Kimble, 1998; Hillery, 2000; Milburn and Braunstein, 1999; Ralph, 2000). Consequently the analysis of squeezing phenomenon in quantum optical systems is an important topic.

Here we analyse the quadrature squeezing for the OENBS. To do so, let us define the quadrature operators $\hat{X}$ and $\hat{Y}$ as follows,

$$\hat{X}_1 = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger) \quad \text{and} \quad \hat{Y}_1 = \frac{1}{\sqrt{2}i}(\hat{a} - \hat{a}^\dagger) \tag{10}$$

and their variances,

$$\mathrm{Var}(\hat{X}_1) = \left\langle \hat{X}_1^2 \right\rangle - \langle \hat{X}_1 \rangle^2 \quad \text{and} \quad \mathrm{Var}(\hat{Y}_1) = \left\langle \hat{Y}_1^2 \right\rangle - \langle \hat{Y}_1 \rangle^2 \tag{11}$$

obey Heisenberg's uncertainty relation,

$$\mathrm{Var}(\hat{X}_1)\mathrm{Var}(\hat{Y}_1)\frac{1}{4} \tag{12}$$

If either $\mathrm{Var}(\hat{X}_1)$ or $\mathrm{Var}(\hat{Y}_1)$ is less than $1/2$, squeezing occurs. Thus, the variances of $\hat{X}_1$ and $\hat{Y}_1$ can be written as,

$$\mathrm{Var}(\hat{X}_1) = \frac{1}{2} + (\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a}^2 \rangle - 2\langle \hat{a} \rangle^2) \quad \text{or} \quad \mathrm{Var}(\hat{Y}_1) = \frac{1}{2} + (\langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^2 \rangle) \tag{13}$$

Using Eqs. (7), (9), and (13) we can investigate the squeezing properties. From Eq. (13) it is apparent that squeezing does not occur in the quadrature $\hat{X}_1$ since $\langle \hat{a} \rangle = 0$.

Figure 1 depicts plots of $V(\hat{Y}_1) = \mathrm{Var}(\hat{Y}_1) - 1/2$ against η for different M and k. As can be seen, the variance is insensitive to M when $|\eta|$ is large enough, and there also exists a critical point η_c. When $\eta > \eta_c$ squeezing exists, and the value of η_c increases as M increases. As k increases, keeping M fixed the squeezing parameters $V(\hat{Y}_1)$ starts at higher values and the squeezing range becomes narrow, as the value of η_c moves to higher values close to 1.

3.2. Amplitude Squared Squeezing (ASS)

Here we discuss another nonclassical effect, namely, the higher order squeezing. This type of squeezing is known as amplitude squared squeezing (ASS) that has been introduced by Hillery (1989). ASS arises in a natural way in a second-harmonic generation. In order to examine whether or not the OENBS exhibit ASS

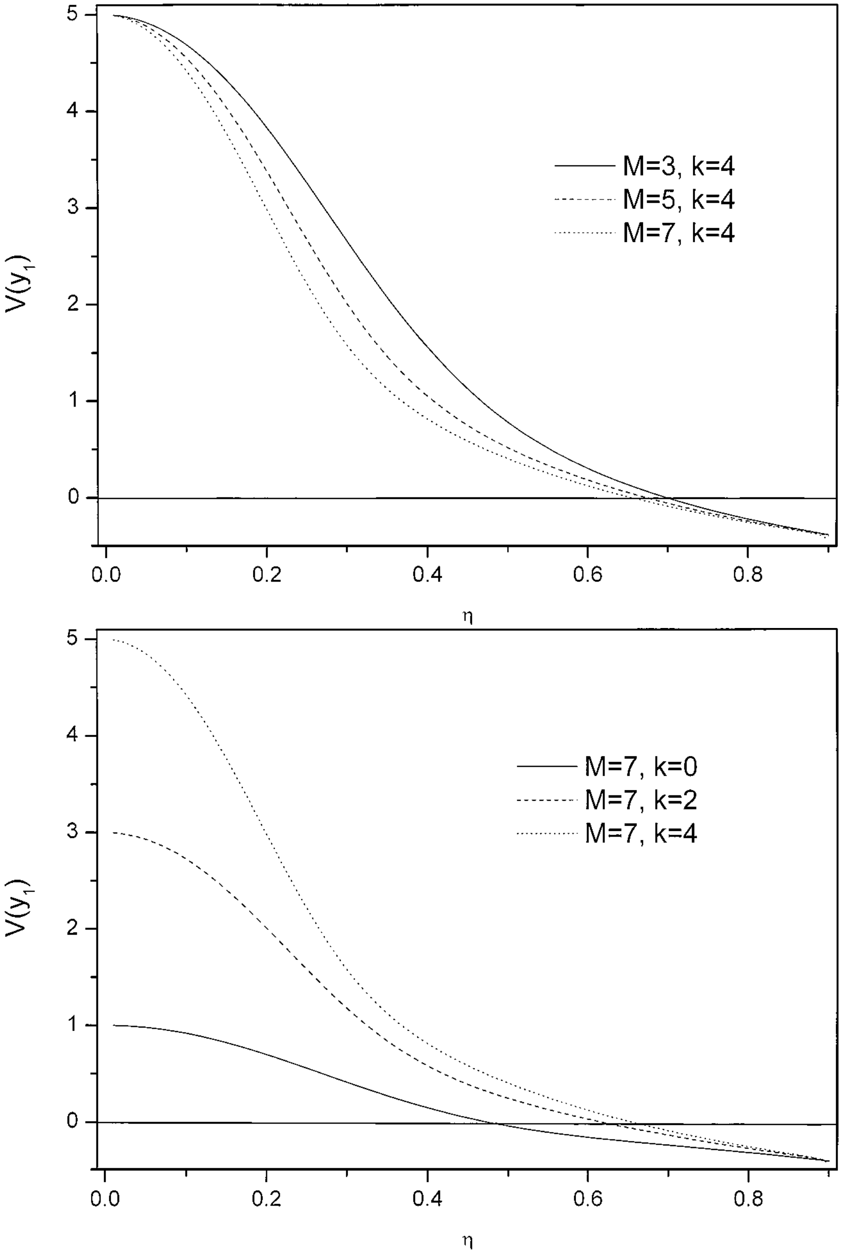


Fig. 1. (a) Quadrature variance against the parameter η for different values of M (k = 4); (b) quadrature variance against the parameter η for different values of k (M = 7).

we introduce the following hermitian operators:

$$\hat{X}_2 = \frac{1}{2}(\hat{a}^2 + \hat{a}^{\dagger 2}) \quad \text{and} \quad \hat{Y}_2 = \frac{1}{2i}(\hat{a}^2 - \hat{a}^{\dagger 2}) \tag{14}$$

then $\hat{X}_2$ and $\hat{Y}_2$ obey the uncertainty relation,

$$\left(\Delta \hat{X}_2^2\right)\left(\Delta \hat{Y}_2^2\right)\frac{1}{4}|\langle[\hat{X}_2, \hat{Y}_2]\rangle|^2 \tag{15}$$

where the variance,

$$\left(\Delta \hat{X}_2^2\right) = \left\langle \hat{X}_2^2 \right\rangle - \langle \hat{X}_2 \rangle^2 \quad \text{and} \quad \left(\Delta \hat{Y}_2^2\right) = \left\langle \hat{Y}_2^2 \right\rangle - \langle \hat{Y}_2 \rangle^2$$

The field is said to be in an amplitude-squared squeezing state if,

$$\left(\Delta \hat{X}_2^2\right) \quad \text{or} \quad \left(\Delta \hat{Y}_2^2\right) < \frac{1}{2}|\langle[\hat{X}_2, \hat{Y}_2]\rangle| \tag{16}$$

The squeezing conditions in (16) can be reduced to the following forms:

$$S_1 = \langle \hat{a}^{\dagger 4} \rangle + \langle \hat{a}^{\dagger 2}\hat{a}^2 \rangle - \langle \hat{a}^2 \rangle < 0$$

or

$$S_2 = \langle \hat{a}^{\dagger 4} \rangle - \langle \hat{a}^{\dagger 2}\hat{a}^2 \rangle < 0 \tag{17}$$

Combining Eqs. (8), (9), and (17) we can study the squeezing effects of OENBS.

Figure 2 is the plot of S_2 against η for different M and k. As seen in Fig. 2(a) and (b) the ASS is sensitive to M and k. When keeping M constant and varying k, we see that the states with higher k show stronger ASS than those with lower values of k. However, squeezing appears after a certain value of η. The same behavior is exhibited when we fix k and vary M. Stronger ASS can be observed for higher values of M.

3.3. Sub-Poissonian Behavior

We turn our attention now to the Glauber second-order zero-time autocorrelation function $g^{(2)}(0)$ which is defined as,

$$g^{(2)}(0) = \frac{\langle \hat{a}^{\dagger 2}\hat{a}^2 \rangle}{\langle \hat{a}^{\dagger}\hat{a} \rangle^2} + \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n}^2 \rangle} \tag{18}$$

This quantity measures the deviation from the Poisson distribution that corresponds to the coherent state with $g^{(2)}(0) = 1$. If $g^{(2)}(0) < 1(>1)$, the field is called sub (super)-Poissonian.

From Eqs. (7), (9), and (18) we can calculate $g^{(2)}(0)$. Figure 3 shows a plot of $g^{(2)}(0)$ against η for different M and k. It can be seen that whatever k or M in the OENBS, these states start sub-Poissonian for $\eta = 0$ with the value $1 - 1/k'$, where k' is the first odd number k, then $g^{(2)}(0)$ increases and becomes

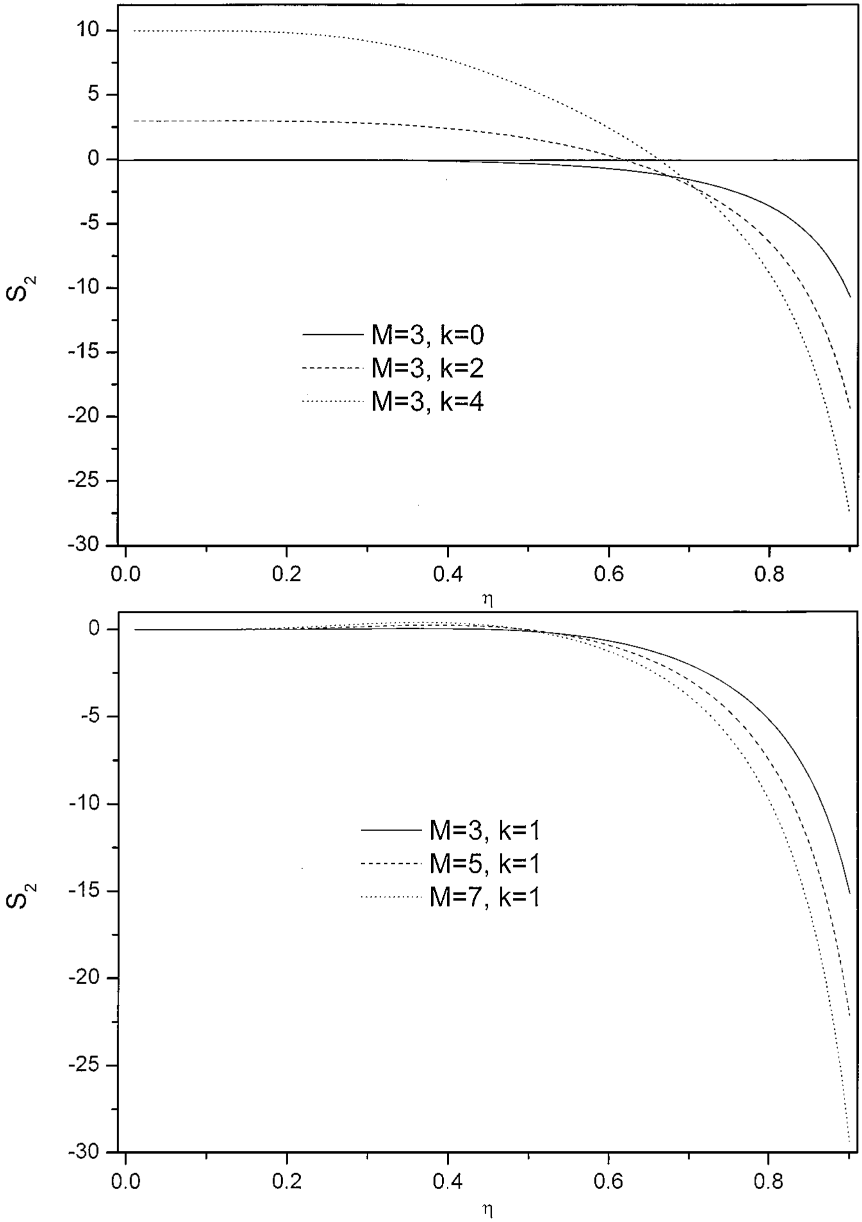


Fig. 2. (a) Amplitude-squared squeezing for different values of k ($M = 3$); (b) amplitude-squared squeezing for different values of M ($k = 1$).

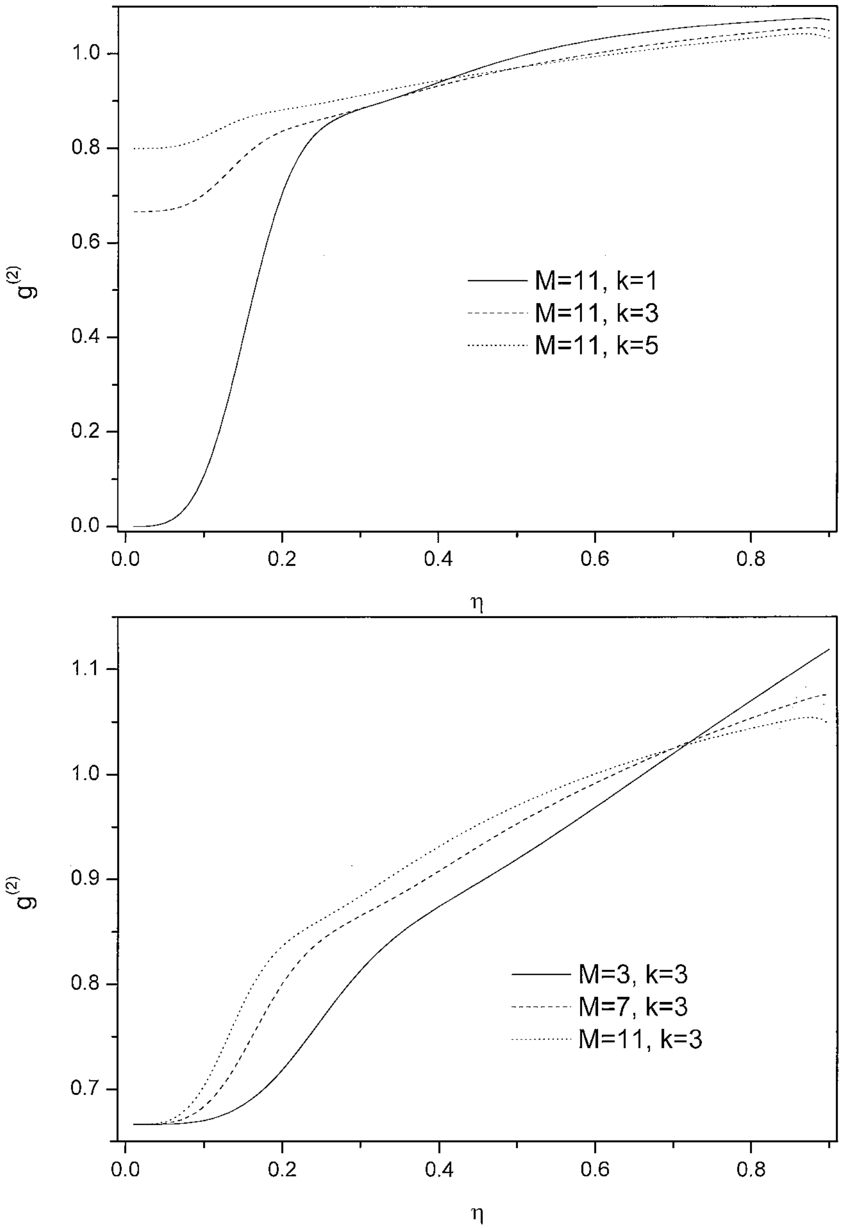


Fig. 3. (a) $g^{(2)}$ Parameter as a function of η for different values of k (M = 11); (b) $g^{(2)}$ parameter as a function of η for different values of M (k = 3); (c) $g^{(2)}$ Parameter as a function of η for different of k (M = 7).

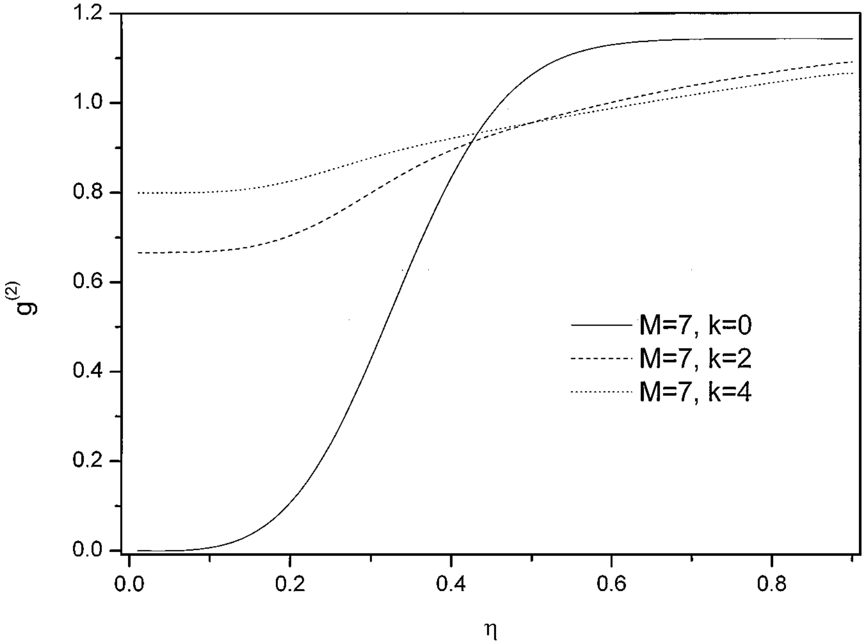


Fig. 3. (Continued.)

super-Poissonian as η takes certain value depending on M and k as shown in Fig. 3(a)–(c).

4. QUASI-PROBABILITY FUNCTIONS

In the following we shall examine the representation of OENBS by the quasi-probability phase space distribution functions. All of these are related to the density matrix, which provides a complete statistical description of the system. There are three types of the quasi-probability functions: Wigner W-, Glauber P-, and Husimi Q-functions. These functions could be used also to describe the nonclassical effects of the system: for example one can employ the negative values of the W-function, the stretching of the Q-function and the high singularities in the P-function as signatures of nonclassical effects. Furthermore, these functions are now accessible to measurements (Leohardt, 1997). Here, we shall consider only the W- and Q-functions. For finding the Wigner function in case of OENBS we consider only the diagonal terms in its representation. We find that,

(i) For even k

$$W(\alpha) = -\frac{2}{\pi}\left|N_o^-\right|^2 e^{-2|\alpha|^2} \sum_{n=0}^{\infty} \left|B_{2n+1}^M(2k_1)\right|^2 L_{2n+2k_1+1}(4|\alpha|^2) \tag{19a}$$

(ii) For odd k

$$W(\alpha) = -\frac{2}{\pi}\left|N_o'^-\right|^2 e^{-2|\alpha|^2} \sum_{s=0}^{\infty} |B_{2s}^M(2k_2+1)|^2 L_{2s+2k_2+1}(4|\alpha|^2) \tag{19b}$$

where $\alpha = x + iy$ and $L_r(z)$ represents the Laguerre polynomial of order r defined by,

$$L_r(z) = \sum_{i=0}^{r}(-1)^i \frac{(r)!z^i}{(i!)^2(r-i)!} \tag{20}$$

As it can be seen from the definition (19) the function $W(\alpha)$ takes negative values for $\alpha = 0$.

On the other hand, the Q-function is positive definite at any point in the phase space for any quantum state and can be written as,

$$Q(\alpha) = \pi^{-1}\langle\alpha|\hat{\rho}|\alpha\rangle \tag{21}$$

In order to find the Q-function, we shall use the definition for the coherent state $|\alpha\rangle$ in the form,

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{l=0}^{\infty}\frac{\alpha^l}{\sqrt{l!}}|l\rangle \tag{22}$$

and the density matrix has the form,

(i) For even k

$$\hat{\rho}_o = |2k_1, \eta, M\rangle_o^{-}\,{}_o^{-}\langle 2k_1, \eta, M|$$

(ii) For odd k

$$\hat{\rho}_o = |2k_2+1, \eta, M\rangle_o^{-}\,{}_o^{-}\langle 2k_2+1, \eta, M|$$

Thus the Q-function is found to have the following expressions,

(i) For even k

$$Q_o(\alpha) = \frac{\left|N_o^-\right|^2}{\pi} e^{-|\alpha|^2}|C_1|^2 \tag{23a}$$

where

$$C_1 = \sum_{n=0}^{\infty} B_{2n+1}^M(2k_1)\frac{(|\alpha|)^{2n+2k_1+1}}{\sqrt{(2n+1)!}}$$

(ii) For odd k

$$Q_o(\alpha) = \frac{\left|N_o'^{-}\right|^2}{\pi} e^{-|\alpha|^2} |C_2|^2 \tag{23b}$$

where

$$C_2 = \sum_{s=0}^{\infty} B_{2s}^{M}(2k_2 + 1) \frac{(|\alpha|)^{2s+2k_2+1}}{\sqrt{(2s)!}}$$

In Fig. 4, the Wigner function of the OENBS is plotted by numerical calculation of Eq. (19) for different values of M, k, and η. It can be seen that the negative part of the Wigner function is quite pronounced especially at the origin. When η is small ($\eta = 0.1$), $M = 5$ and changing of k, we see that at $k = 1$, $W(\alpha)$ has central peak with a crater-like structure around it, Fig. 4(a). When k starts to take values larger than before, see Fig. 4(b) and (c), the central peak is surrounded by wobbling circles due to the contribution of higher excitations. The existence of the oscillations associated with the negative quasi-probability values, is a signature of the nonclassical state. However, the widths of the oscillations are much broader as compared to Fig. 4(a). Numerical calculations show that $W(\alpha)$ is insensitive to changes in M, However, $W(\alpha)$ is very sensitive to the choice of k and η.

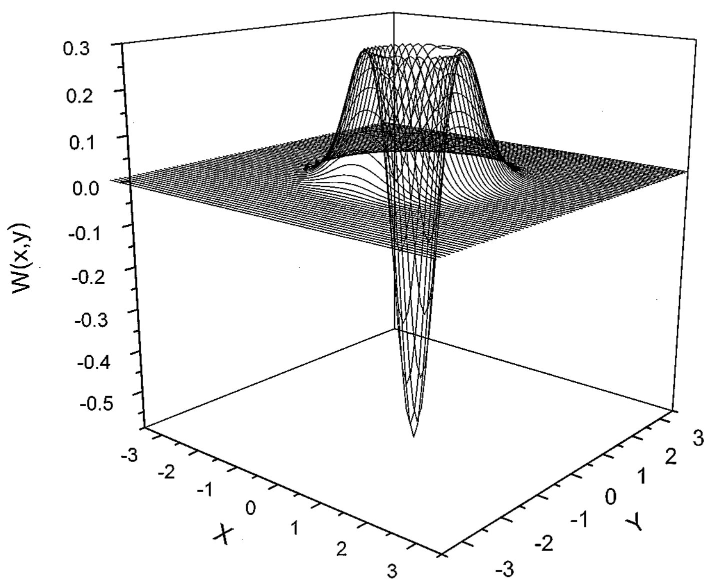


Fig. 4. (a) Wigner function for M = 5, k = 1, and η = 0.1; (b) wigner function for M = 5, k = 4, and η = 0.1; (c) wigner function for M = 5, k = 2, and η = 0.6.

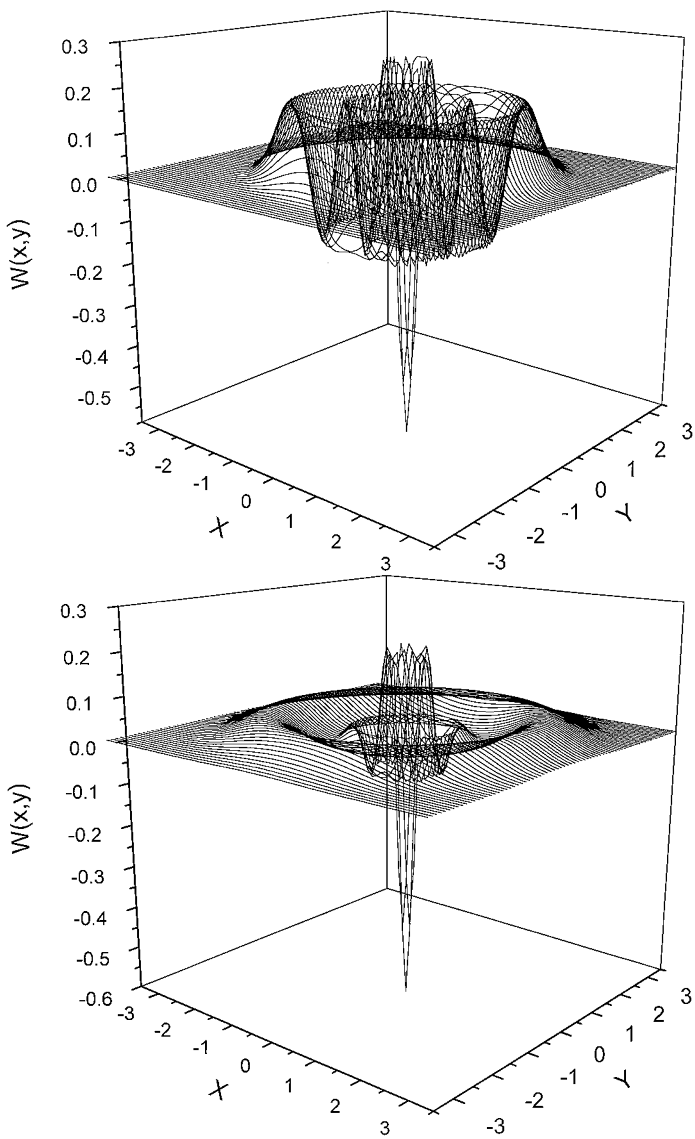


Fig. 4. (Continued.)

With respect to $Q_o(\alpha)$ for OENBS, it is plotted for different values of M, k, and η (see Fig. 5). When η is small ($\eta = 0.1$) the state $|2, 0.1, M\rangle_o^-$ of the Q-function is insensitive to changes in M [compare Fig. 5(a) and (b)] where the state $|3\rangle$ is the most effective state and its contribution is the mainly effective one. Also,

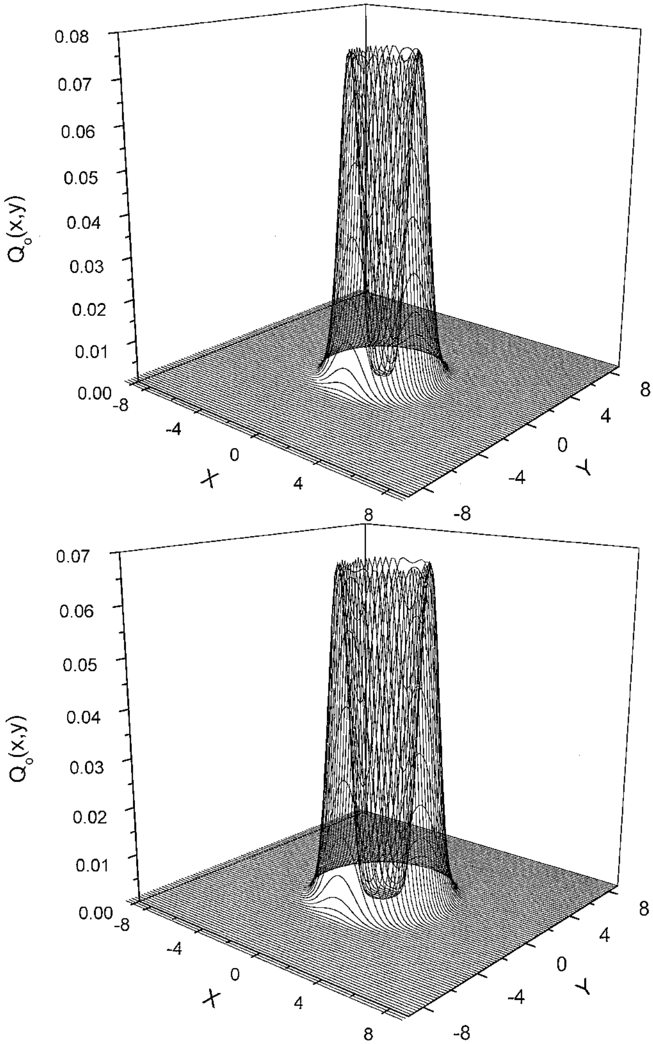
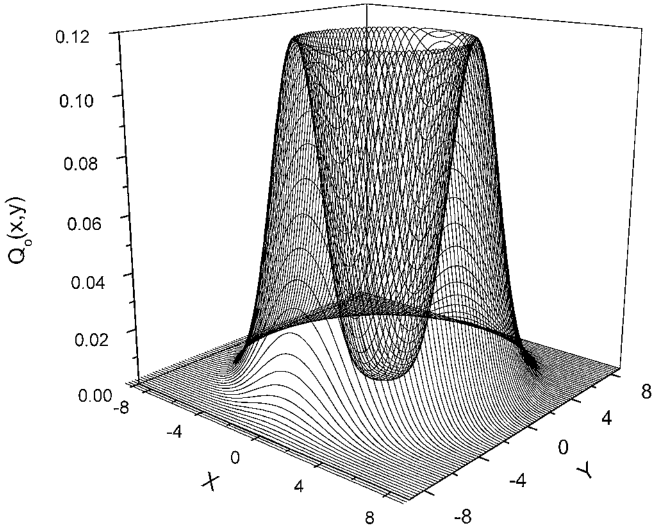


Fig. 5. (a) Q-function for M = 5, k = 2, and η = 0.1; (b) Q-function for M = 11, k = 2, and η = 0.1; (c) Q-function for M = 5, k = 0, and η = 0.9; (d) Q-function for M = 5, k = 2, and η = 0.9.

we find the center-crater-like spreads out in the phase space as η increases. When η increases ($\eta = 0.9$) and $M = 5$ more states come to affect the Q-function, which means that its diameter increases as the number of states increases (see Fig. 5(c) and (d). The shape of the function is sensitive to any change in either k or η as shown in Fig. 5.



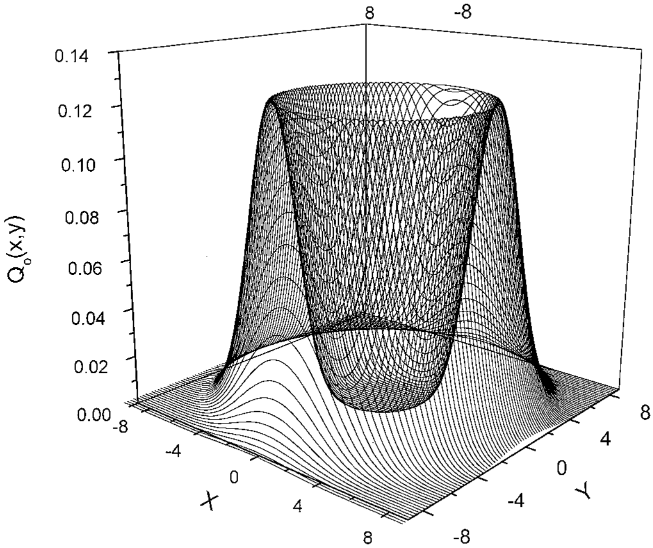


Fig. 5. (Continued.)

5. PHASE PROPERTIES

In fact, there are different techniques for phase description (Lynch, 1995; Tanas *et al.*, 1996) based on a Hermitian quantum phase operator or associated with quasi-probability distribution functions in a phase space. Here we use the formalism of Barnett and Pegg that is based on defining the hermitian phase operator in a finite dimensional phase space (Barnett and Pegg, 1986, 1989a,b). They defined in this space the phase operator as the projection operator on the particular phase state multiplied by the corresponding value of the phase. The main idea in their technique is to evaluate all expectation values of physical variables in a finite-dimensional Hilbert space and then taking the limit to obtain the measured quantities.

In this approach, in the $(s+1)$ dimensional subspace, one can choose the following $(s+1)$ orthonormal phase states as bases,

$$|\theta_m\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle \tag{24}$$

where $|n\rangle$ are the number states and θ_m is given by,

$$\theta_m = \theta_0 + \frac{2\pi m}{s+1}; \quad m = 0, 1, \ldots, s \tag{25}$$

the value of θ_0 is arbitrary, and is taken here to be zero.

The Hermitian phase operator is then defined by,

$$\hat{\Phi}_\theta = \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m| \tag{26}$$

which has the eigenstate $|\theta_m\rangle$ as its eigenstate with the eigenvalue θ_m. The state of the form,

$$|b\rangle = \sum_{n=0}^{s} b_n\, e^{in\chi} |n\rangle \tag{27}$$

is known as a partial phase state (Pegg and Barnett, 1988, 1989), where b_n are real and positive, and χ is a phase.

The phase probability distribution for the partial phase state is given by,

$$P(\theta_m) = |\langle\theta_m|b\rangle|^2 \tag{28}$$

Since the density of phase state is $(s+1)/2\pi$, then the limit $s \to \infty$ is taken and the continuous-phase probability distribution is introduced by,

$$P(\theta) = \lim_{s\to\infty} \frac{s+1}{2\pi} |\langle\theta_m|b\rangle|^2 \tag{29}$$

We obtain from (27) and (29) the Pegg–Barnett phase probability distribution for the partial coherent state (27) in the form,

$$P(\theta) = \frac{1}{2\pi}\left(1 + 2\sum_{n>m} b_m b_n \cos[(n-m)\theta]\right), \quad -\pi \le \theta \le \pi \tag{30}$$

In case of OENBS, the function $P(\theta)$ has the following form,

(i) For even k

$$P(\theta) = \frac{1}{2\pi}\left(1 + 2\sum_{2n+1>2m+1} b_{2m+1} b_{2n+1} \cos[(2n-2m)\theta]\right) \tag{31a}$$

(ii) For odd k

$$P(\theta) = \frac{1}{2\pi}\left(1 + 2\sum_{2s>2s'} b_{2s'}b_{2s}\ \cos[(2s-2s')\theta]\right) \tag{31b}$$

Since $b_{2n+1} = B^{M}_{2n+1}(2k_1)$ and $b_{2s} = B^{M}_{2s}(2k_2+1)$ are given from Eq. (4).

In Fig. 6 we have plotted the phase distribution of OENBS given by (31) against the parameter η for different values of M and k. From Fig. 6 one can

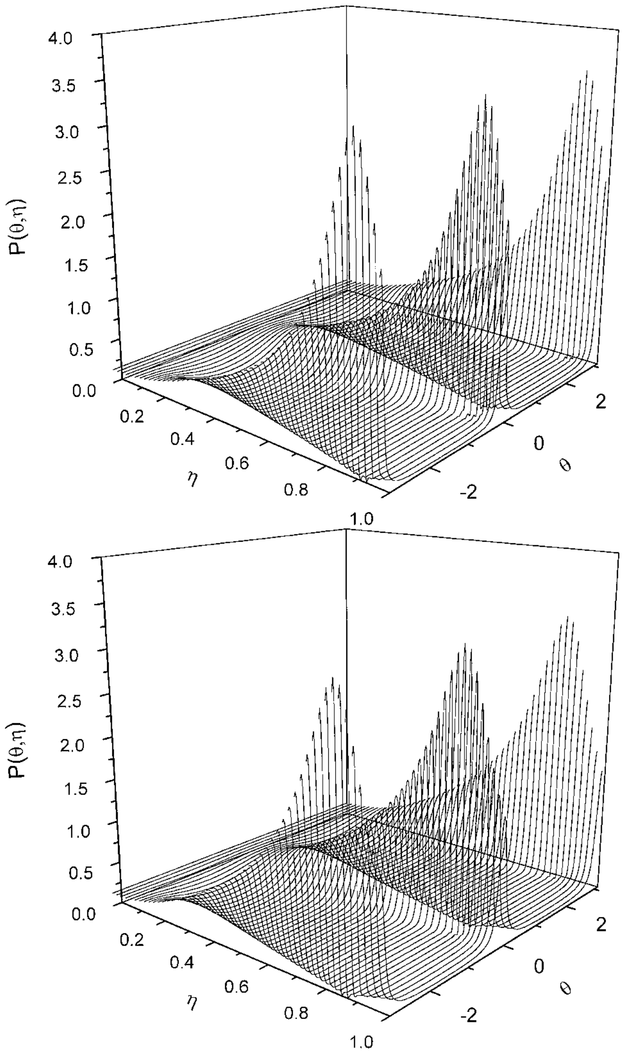


Fig. 6. (a) Phase probability at $M = 5$ and $k = 1$; (b) phase probability at $M = 9$ and $k = 2$.

observe that, the distribution initially approaches the value $1/2\pi$ as $\eta \to 0$. This means that in this limit the phase information is lost. As η increases, we can observe a central peak at $\theta = 0$ and two lateral wings at $\theta \to \pm\pi$ developing. In fact, this behavior has been seen for odd binomial states (El-Orany *et al.*, 1999). Also we note that the peaks are stretching along η-axis, and have maximum values at $0 < \eta < 1$ due to the nature of photon distribution in the state.

6. CONCLUSION

In this paper we have investigated some quantum statistical properties of a new quantum state. This state is called odd excited negative binomial state (this state interpolates between the odd number state and the odd excited coherent state). The squeezing properties have been studied in detail, and it is found that there exist critical point for the parameter η, for which the squeezing is exhibited as $\eta > \eta_c$. Glauber second-order correlation function $g^{(2)}(0)$ has been discussed. The distribution starts at $\eta = 0$ sub-Poissonian whatever the value of M and k. Later as η increases the distribution changes from sub-Poissonian to super-Poissonian. We have also discussed the quasi-probability function W-function, and Q-function. Nonclassical effects have been observed for careful choice of the parameters. A negative value for the Wigner function is a signature for nonclassical effects of the state. Also the shape of W- and Q-functions are very sensitive to any change in either k or η. Finally the phase distribution function in the sense of Pegg–Barnett has been calculated and investigated. The phase distribution has one central peak at $\theta = 0$ and two lateral wings at $\theta \to \pm\pi$ for $0 < \eta < 1$. This behavior has been exhibited for earlier studies.

REFERENCES

Agarwal, G. S. (1992). *Physical Review A* **45**, 1787.

Agarwal, G. S., and Inguva, R. (1991). *Quantum Optics*, Plenum, New York.

Barnett, S. M. (1998). *Journal of Modern Optics* **45**, 2201.

Barnett, S. M., and Pegg, D. T. (1986) *Journal of Physics Mathematics* **19**, 3849.

Barnett, S. M., and Pegg, D. T. (1989a). *Physical Review A* **39**, 1665.

Barnett, S. M., and Pegg, D. T. (1989b). *Journal of Modern Optics* **36**, 7.

Batarfi, H., Abdalla, M. S., Obada, A.-S. F., and Hassan, S. S. (1995). *Physical Review A* **51**, 2644.

Braunstein, S. L., and Kimble, H. J. (1998). *Physical Review Letters* **80**, 869.

El-Orany, F. A. A., Mahran, M. H., Obada, A.-S. F., and Abdalla, M. S. (1999). *International Journal of Theoretical Physics* **38**, 1493.

Fu, H.-C., and Sasaki, R. (1997a). *Journal of the Physical Society of Japan* **66**, 1989.

Fu, H.-C., and Sasaki, R. (1997b). *Journal of Mathematical Physics* **38**, 3968.

Hillery, M. (1989). *Physical Review A* **40**, 3147.

Hillery, M. (2000). *Physical Review A* **61**, 022309.

Joshi, A., and Lawande, S. V. (1991). *Journal of Modern Optics* **38**, 2009.

Joshi, A., and Obada, A.-S. F. (1997). *Journal of Physics A: Mathematical and General* **30**, 81.

Kendall, M., and Stuart, A. (1977). *The Advanced Theory of Statistics*, Vol. 1, Griffin, London, chap 5.
Leohardt, U. (1997). *Measuring the Quantum State of Light*, Cambridge University Press, Cambridge, U.K.
Lynch, R. (1995). *Physical Reports* **250**, 367; See the special issue of *Physica Scripta* **T48** (1993).
Mahran, M. H., and Obada, A.-S. F. (1988). *Journal of Modern Optics* **35**, 1847.
Matsuo, K., 1990. *Physical Review A* **41**, 519.
Milburn, G. J., and Braunstein, S. L. (1999). *Physical Review A* **60**, 937.
Obada, A.-S. F., Hassan, S. S., Puri, R. R., and Abdalla, M. S. (1993). *Physical Review A* **48**, 3174.
Pegg, D. T., and Barnett, S. M. (1988). *Europhysics Letters* **6**, 483.
Pegg, D. T., and Barnett, S. M. (1989). *Physical Review A* **39**, 1665.
Ralph, T. C. (2000). *Physical Review A* **61**, 010303(R).
Simon, R., and Satyanarayana, M. V. (1988). *Journal of Modern Optics* **35**, 719.
Srinivasan, R., and Lee, C. T. (1996). *Physics Letters A* **218**, 151.
Stoler, D., Saleh, B. E. A., and Teich, M. C. (1985). *Optica Acta* **32**, 345.
Tanas, R., Miranowicz, A., and Gantsog, Ts. (1996). *Progress in Optics*, Vol. 35, Elsevier, Amsterdam.
Vourdas, V., and Bishop, R. F. (1995). *Physical Review A* **51**, 2353.
Wang, X.-G., and Fu, H.-C. (2000). *International Journal of Theoretical Physics* **39**, 1437.